The anti-decuplet candidate $\Xi^{--}(1862)$ as a heptaquark with the overlap of two anti-kaons and a nucleon

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I study the very recently discovered S=-2 resonance Ξ^{--} (1886) by the NA49 collaboration at the CERN SPS. This resonance was already predicted, with a mass close to 1.9 GeV, in a recent publication mostly dedicated to the S=1 resonance $\Theta^+(1540)$. To confirm this recent prediction, I apply the same standard quark model with a quark-antiquark annihilation constrained by chiral symmetry. This method also explained with success the repulsive hard core of nucleon-nucleon, kaon-nucleon exotic scattering, and the short range attraction present in pion-nucleon and pion-pion non-exotic scattering. I find that repulsion excludes the Ξ^{--} as a $ddss\bar{u}$ s-wave pentaquark. I explore the Ξ^{--} as a heptaquark, equivalent to a $N+K+K$ linear molecule, with positive parity and total isospin $I=3/2$. I find that the kaon-kaon repulsion is cancelled by the attraction existing in the kaon-nucleon channels. In our framework this state is easier to bind than the Θ^+ described by a $\pi+K+N$ borromean bound-state. The remaining $I=1/2$ doublet and $I=1$ triplet of the exotic anti-decuplet are also studied, and the coupling to p-wave decay channels is briefly addressed.

I. INTRODUCTION

In this paper I study the exotic hadron Ξ^{--} (narrow hadron resonance of 1862 MeV decaying into a $\Xi^-\pi^-$) very recently discovered by the NA49 collaboration at the CERN SPS [1, 2, 3]. The simultaneous discovery of a resonance with similar mass and width decaying into a $\Xi^-\pi^+$ provides evidence for a $I=3/2$ iso-quadruplet. This completes a missing link of the anti-decuplet [4, 5, 6] which includes the recently discovered $\Theta^+(1540)$ [7, 8, 9, 10, 11]. Moreover pentaquark structures have been observed in the lattice both with parity + and with parity - [12, 13, 14].

The Θ^+ is an extremely exciting state, because it may be the first exotic hadron to be discovered, with quantum numbers that cannot be interpreted as a quark and an anti-quark meson or as a three quark baryon. In what concerns the quark structure it was known for a long time that the simplest and lightest s-wave multiquarks would be repulsive and very unstable. For instance we recently computed [15] the mass of the groundstate s-wave $I=0$, $J^P=1/2^+$ $uudd\bar{s}$ pentaquark, and we checked that it would have a mass of 1535 MeV, close to M_{Θ^+}. However in this channel we find a purely repulsive exotic $N-K$ hard core s-wave interaction [16, 17, 18]. This suggests why pentaquarks have been hard to find, and at the same time this indicates that pentaquarks should include an excitation. Because excited multiquark systems are indeed difficult to study, different perspectives of the pentaquarks are welcome to fully address these new states.

Exotic multiquarks are expected since the early works of Jaffe [19, 20, 21, 22]. Soon after the Θ^+ was observed, Jaffe and Wilczek [23], and Karliner and Lipkin [24] proposed that the pentaquarks are arranged in microscopic coloured diquarks or triquarks, connected by a string in a p-wave state. This is a very appealing structure, in particular the p-wave system tends to have a narrow width because the decay is only possible if the diquarks overlap. This model needs a novel cancellation of some of the mass of diquarks, to compensate the large p-wave excitations.

The exotic anti-decuplet was first predicted, with the correct Θ^+ mass and similar decay width, by Diakonov, Petrov and Polyakov [6]. These authors interpret the exotic anti-decuplet as a rotational excitation of the chiral topological soliton [4, 5, 6]. This approach suggests that chiral symmetry and p-wave excitations are crucial to understand the pentaquarks. However the Skyrme Chiral Soliton has some difficulty to reproduce the short range repulsion in N-N interactions.

Soon after the Θ^* was discovered, we proposed another pentaquark model [15] which estimates a mass of 1.9 Gev for the S= -2, Q = -2 state. Importantly, this predicted mass is quite close to the observed one. To motivate our proposal let me first review the five possible excitations of the quark model, in decreasing energy shift order. The first excitation is the radial one, with an energy shift of $M_{\rho^*(1^{--})}-M_{\rho(1^{--})}\simeq 700$MeV. The second excitation is the angular one, with an energy shift of $M_{f_1^{(1^{++})}}-M_{\rho^{(1^{--})}}\simeq 500$MeV. The third excitation is the spin one, with an energy shift of $M_{K^{*(1^{--})}}-M_{K^{(0^{-+})}}\simeq 400$MeV. The fourth excitation is the flavour one, with an energy shift of $M_{K^{*(1^{--})}}-M_{\rho^{(1^{--})}}\simeq 150$MeV or $M_{\omega^{(1^{--})}}-M_{\rho^{(1^{--})}}\simeq 10$MeV. The fifth excitation is the quark-antiquark pair creation one, with an energy shift of $M_{\pi^{(0^{-+})}}\simeq 140$MeV. The light mass $M_{\theta^+}\simeq M_N+M_K+100$MeV of the θ^+ pentaquark suggests that it either has a flavour excitation or a $q-\bar{q}$ pair creation, and not a p-wave excitation. Because the production processes show no evidence for a weaker flavour changing reaction, we explored [15] the $q-\bar{q}$ creation hypothesis. Moreover, when a flavor singlet quark-antiquark pair $u\bar{u}+d\bar{d}$ is created in the pentaquark H, the resulting crypto-heptaquark H' is a state with an opposite par-

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ity to the original H, where the reversed parity occurs due to the intrinsic parity of fermions and anti-fermions. In this sense the new heptaquark H' can be regarded as the chiral partner of H. And, because H' is expected to have the lowest possible mass, it is naturally rearranged in a baryon belonging to the s-wave baryon octet and in two pseudoscalar mesons belonging to the s-wave meson octet. In this approach the mass of the heptaquark H' is simply expected to be slightly lower than the exact sum of these standard hadron masses due to the negative binding energy. For instance we recently suggested [15] that the Θ^+ is probably a $K-\pi-N$ molecule with binding energy of 30 MeV, a borromean three body s-wave boundstate of a π, a N and a K [15, 25, 26], with positive parity [27] and total isospin $I=0$. We also addressed the S= -2, Q = -2 state Ξ^{--}, suggesting that it is a $\bar{K}-N-\bar{K}$ molecule. The NA49 result for the iso-quadruplet of 1.862 GeV is consistent with a binding energy of 60 MeV for the hadronic molecule.

In this paper I extend the quantitative techniques used in our first publication for the Θ^+, [15] to the remaining of the anti-decuplet. I start by reviewing, in Section II, the standard Quark Model (QM), and the Resonating Group Method (RGM) [28, 29] which is adequate to study states where several quarks overlap. Using the RGM, I show that the corresponding exotic baryon-meson short range s-wave interaction is repulsive in exotic channels and attractive in the channels with quark-antiquark annihilation. In most iso-multiplets, except in the $S=-1$ iso-multiplet the short range repulsion contradicts a possible pentaquark with a narrow width. Section III proceeds with the study of the heptaquarks in the exotic anti-decuplet, which are bound by the attractive non-exotic $\bar{K}-N$ and $\bar{K}-K$ interactions. I compute the masses of the exotic anti-decuplet. In Section IV the coupling and decays to p-wave channels are addressed. Finally the conclusion is presented in Section V.

II. FRAMEWORK

Our Hamiltonian is the standard QM Hamiltonian,

$$H=\sum_i T_i+\sum_{i<j}V_{ij}+\sum_{i\bar{j}}A_{i\bar{j}} \qquad (1)$$

where each quark or antiquark has a kinetic energy T_i with a constituent quark mass. The colour dependent two-body interaction V_{ij} includes the standard QM confining term and a hyperfine term. The quark-antiquark annihilation-creation potential $A_{i\bar{j}}$ is necessary when the potential complies with chiral symmetry, including the light pion mass and the Adler Zero [30, 31, 32, 33]. The RGM provides an accurate framework [28, 29] to compute the effective multiquark energy using the matrix elements of the quark-quark interactions. Any multiquark state can be decomposed in antisimmetrised combinations of simpler colour singlets, the baryons and mesons. For the purpose of this paper the details of the potentials in eq. (1) are unimportant, only their matrix elements, extracted from the baryon spectroscopy, matter

$$\begin{aligned}\langle V_{hyp}\rangle &\simeq \frac{4}{3}\left(M_\Delta-M_N\right)\ ,\\ \langle A\rangle_{S=0} &\simeq -\frac{2}{3}\left(2M_N-M_\Delta\right)\ .\end{aligned} \qquad (2)$$

The detailed calculations are similar to the ones in reference [15], and lead to the attraction/repulsion criterion,
- *whenever the two interacting hadrons have quarks (or antiquarks) with a common flavour, the repulsion is increased by the Pauli principle,*
- *when the two interacting hadrons have a quark and an antiquark with the same flavour, the attraction is enhanced by the quark-antiquark annihilation.*

In the particular case of one nucleon interacting with anti-kaons and with kaons, this implies that the short range $K-N$ and $\bar{K}-\bar{K}$ interactions are repulsive, while the short range $\bar{K}-K$ and $\bar{K}-N$ interactions are attractive. Quantitatively [15, 34, 35], the effective potentials computed for the channels relevant to this paper, are

$$\begin{aligned}V_{K-N} &= \frac{\frac{1}{4}+\frac{1}{6}(\vec{\tau}_K+\vec{\tau}_N)^2}{1-\frac{1}{6}(\vec{\tau}_K+\vec{\tau}_N)^2}\langle V_{hyp}\rangle|\phi^\alpha_{\mathbf{0}}><\phi^\alpha_{\mathbf{0}}|\ ,\\ V_{K-\bar{K}} &= \frac{2+\sigma-(\vec{\tau}_K+\vec{\tau}_{\bar{K}})^2}{6}\langle A\rangle\,|\phi^\alpha_{\mathbf{0}}><\phi^\alpha_{\mathbf{0}}|\ ,\\ V_{\bar{K}-N} &= \frac{3-(\vec{\tau}_{\bar{K}}+\vec{\tau}_N)^2}{6}\langle A\rangle\,|\phi^\alpha_{\mathbf{0}}><\phi^\alpha_{\mathbf{0}}|\ ,\\ V_{\bar{K}-\bar{K}} &= \frac{\frac{1}{4}(\vec{\tau}_{\bar{K}}+\vec{\tau}_{\bar{K}})^2}{1-\frac{1}{12}(\vec{\tau}_{\bar{K}}+\vec{\tau}_{\bar{K}})^2}\langle V_{hyp}\rangle|\phi^\alpha_{\mathbf{0}}><\phi^\alpha_{\mathbf{0}}|\ ,\end{aligned} \qquad (3)$$

where $\vec{\tau}$ are the isospin matrices, normalised with $\vec{\tau}^{\,2}=\tau(\tau+1)$. In the chiral limit one would expect that the $I=0\,, V_{\bar{K}-N}$ cancels with the $I=1$, $V_{\bar{K}-\bar{K}}$, and this is confirmed by eq. (3). To arrive at eq.3 we used a harmonic oscillator basis $|\phi^\alpha_{\mathbf{n}}>$ for the multiquark wavefunction , where the inverse hadronic radius α is the only free parameter in this framework.

III. BINDING IN THE ANTI-DECUPLETS

The simplest pentaquarks are not expected to bind due to the attraction/repulsion criterion. For instance the Ξ^{--} cannot be a $dds\bar{s}u$ pentaquark. The possible elementary color singlets $(dds)-(s\bar{u})$ or $(dss)-(d\bar{u})$ are repelled because the elementary color singlets share the same flavour d or s. This also implies that the $\pi-\Xi$ and $\bar{K}-\Sigma$ systems are unbound. Then the only way to have attraction consists in adding at least one quark-antiquark pair to the system.

However including an extra pion in the fundamental configurations is not possible, except in the Θ^+. Because the pion is very light it is not expected to bind into a narrow resonance [36], except in the Θ^+ where it may be attracted both by a K and a N in a borromean structure. Moreover the K and N are repelled in this framework,

and the narrow Θ^+ is not supposed to exist unless it includes a pion to bind it.

Nevertheless, in the other iso-multiplets, where a $\bar{K}$ exits, binding is possible because the $\bar{K}$ is attracted both by the K and the N. Therefore, although the $\bar{K}-\bar{K}$ system is repulsive, the $K-\bar{K}-N$, $\bar{K}-N$ and $\bar{K}-N-\bar{K}$ systems are expected to bind. It is then convenient to build the anti-decuplet like a combinatoric Newton pyramid, starting by the summit, $uudd\bar{s}$, I=0, Θ^+. To reach any of the other three iso-multiplets in the anti-decuplet, one simply needs to add respectively one, two or three $I=1/2$, $\bar{K}$, ($s\bar{u}$, $s\bar{d}$) to the Θ^+. Altough we advocate [15] that Θ^+ is a $K-\pi-N$ linear molecule, let us consider for the flavour purpose, that it has the flavour of a I=0, $K-N$ system.

The next $I=1/2$, N^* iso-multiplet (uud , udd) can be obtained combining a $I=1/2$, $\bar{K}$, ($s\bar{u}$, $s\bar{d}$) with the Θ^+. A possible binding structure is a $K-\bar{K}-N$ linear molecule.

The next $I=1$, Σ^* iso-multiplet (uus , uds , dds) can be obtained combining a $I=1/2$, $\bar{K}$, ($s\bar{u}$, $s\bar{d}$) with the N^*. The simplest exotic binding structure is pentaquark with the flavour of a $\bar{K}-N$ system.

Finally the last $I=3/2$, Ξ^* iso-multiplet ($uussd\bar{}$, uss, dss, $ddss\bar{u}$) can be obtained combining a $I=1/2$, $\bar{K}$, ($s\bar{u}$, $s\bar{d}$) with the Σ^*. In this case the simplest exotic binding structure is a $\bar{K}-N-\bar{K}$ linear molecule. The different multiquarks are summarised in Table I.

channel	flavour m_I	multiquark
$I=0\,, S=1$	$uudd\bar{s}$	$K-\pi-N$ molecule
$I=\frac{1}{2}\,, S=0$	$uud\,, udd$	$K-\bar{K}-N$ molecule
$I=1\,, S=-1$	$uus\,, uds\,, dds$	$\bar{K}-N$ pentaquark
$I=\frac{3}{2}\,, S=-2$	$uussd\bar{}\,, uss\,, dss\,, ddss\bar{u}$	$\bar{K}-N-\bar{K}$ molecule

TABLE I: Proposed list of multiquark states, candidates to the exotic pentaquark and heptaquark anti-decuplets.

A. The I=1 pentaquark

I now compute the energy of the simplest state, the I=1 iso-triplet Σ^*, as a pentaquark with the quantum numbers of a $\bar{K}-N$ system. The other $\bar{K}-N$ system, the iso-singlet $I=0$ $\Lambda(1405)$ has been studied in detail in the literature. Nevertheless the $I=1$ system is also attractive, moreover the $\bar{K}-N$ binding is relevant to the $I=3/2$, Ξ^*. In this case the reduced mass is $\mu=325$ MeV, and the potential $v|\phi^\alpha_{000}\rangle\langle\phi^\alpha_{000}|$ is attractive and separable, where $v=-71.5MeV$ was computed in eq. (3) and where α is a free parameter. In the $\bar{K}-N$ case, binding exists if $\alpha<\sqrt{-4\mu\, v}=304$ MeV. However for a binding energy of the order of 60 MeV, close to the one proposed for the the exotic Ξ^{--} and consitent with a crypto-exotic $\Sigma^*(1385)$ a too small $\alpha=74MeV$ would be required. Therefore binding is expected in this system, although an accurate prediction of the binding energy of this Σ^* system would need technical improvements of our method, see Section V for details.

B. The I=1/2 and I=3/2 heptaquarks

I now use an adiabatic Hartree method to study the stability of the linear $I=3/2$ $\bar{K}-N-\bar{K}$ molecule. While the $I=1$ pentaquark is not a molecule since the five quarks and antiquarks overlap in a s-wave state, here the large $\bar{K}-\bar{K}$ repulsion, with $v=234$ MeV, prevents the overlap of all the quarks. Essentially the wave-function of the N is centred between the two $\bar{K}$, and the two $\bar{K}$ only overlap with the nucleon, but not with each other. This results in a linear molecule. I solve a Schrödinger equation for a $\bar{K}$ in the potential produced by a nucleon placed at the origin and by the other $\bar{K}$ placed at a distance $-\mathbf{a}$ of the nucleon. The potential of the N is produced by a $\bar{K}$ anti-kaon at the point $-\mathbf{a}$ and another anti-kaon at the point $+\mathbf{a}$. The potentials are respectively,

$$V_{\bar{K}} = v_{\bar{K}-N}|\phi^\alpha_{\mathbf{0}}><\phi^\alpha_{\mathbf{0}}| + v_{\bar{K}-\bar{K}}|\phi^\alpha_{-\mathbf{a}}><\phi^\alpha_{-\mathbf{a}}|\ , \quad (4)$$

$$V_N = v_{\bar{K}-N}|\phi^\alpha_{-\mathbf{a}}><\phi^\alpha_{-\mathbf{a}}| + v_{\bar{K}-N}|\phi^\alpha_{\mathbf{a}}><\phi^\alpha_{\mathbf{a}}|\ , \quad (5)$$

where the sub-index denotes the position of the potential. This produces three binding energies $E_{\bar{K}}$, $E_{\bar{K}}$, E_N, and three wave-functions. In the Hartree method the total energy is the sum of these energies minus the matrix elements of the potential energies,

$$E=2E_{\bar{K}}+E_N-2<\phi_{\bar{K}}|V_{\bar{K}}|\phi_{\bar{K}}>-<\phi_N|V_N|<\phi_N>\,. \quad (6)$$

This is easily computed once the two Schrödinger equations are respectively solved with the potentials (4) and (5). The total energy is a function of the distance $\mathbf{a}$, and I minimise it as a function of $|\mathbf{a}|$. The energy minimum is obtained for $|\mathbf{a}|\simeq 0.7/\alpha$. The centres of the two $\bar{K}$ are separated by a distance of $1.4/\alpha$, and therefore they essentially do not overlap. I first verify that binding is easy to produce in this system, although an α consistent with the expected nucleon radius ≤ 0.7 Fm would produce a small binding energy. Then the case of the excessively small $\alpha=74MeV$, that provides a large binding for the $\bar{K}-N$ system, is investigated. This would already correspond to an extremely large nucleon mean radius α^{-1}, nevertheless let us check that a larger binding can be obtained for the $\bar{K}-N-\bar{K}$. In this case I find that all the three one hadron binding energies are similar to -40 MeV. When the potential energies are subtracted in eq. (6), the final binding energy results in $E\simeq$ -40MeV which is consistent with a Ξ^{--} mass of $1,88$ GeV. This remains larger than a mass of $1,86$ GeV for the Ξ^{--}, however a complete computation of the effect of coupled channels on the binding energy remain to be estimated. The length $1.4/\alpha$ of the $\bar{K}-N-\bar{K}$ linear molecule is larger than 1 Fm, even when we have a small nucleon, with $\alpha=304$ MeV.

A similar binding energy, and a similar mass of the order of 1.9 GeV can be computed for the $K-\bar{K}-N$ system in the non-exotic iso-doublet.

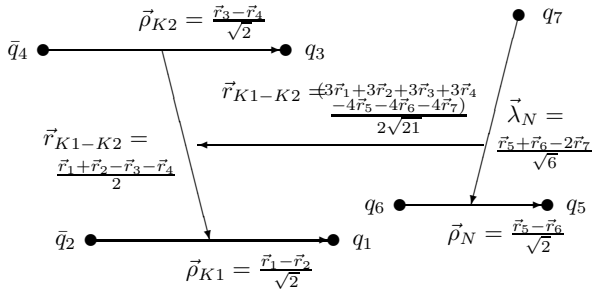

FIG. 1: The Jacobi coordinates of the heptaquark.

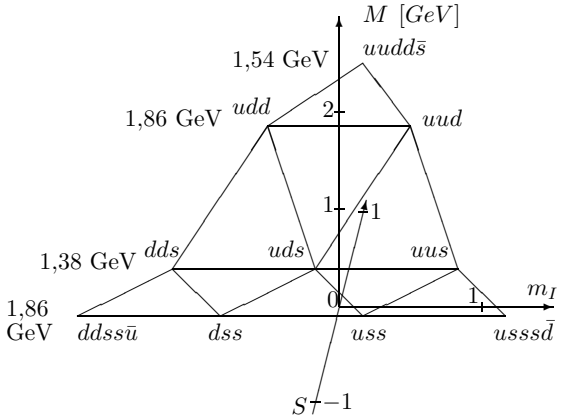

FIG. 2: The masses of the narrow exotic anti-decuplets, in the heptaquark and pentaquark scenarios, are shown in a three dimensional strangeness-flavour-mass plot. The Θ^+ iso-singlet is a $K-\pi-N$ molecule. The iso-doublet N^* is a $K-\bar{K}-N$ molecule. The iso-triplet Σ^* is a $\bar{K}-N$ pentaquark. The Ξ_5^* iso-quadruplet is a $\bar{K}-N-\bar{K}$ molecule.

IV. COUPLING AND DECAYS TO P-WAVE CHANNELS

In the case of the $\Xi^{--}(1860)$, the mass is already 400 MeV higher than the mass of the $\Xi^- + \pi^-$ system, and 200 MeV higher than the mass of the $\Sigma^- + K^-$ system. In this case the p-wave excitation may also be relevant, as suggested by the Chiral Soliton model and by the Diquark model. Moreover the decay channels are also p-wave channels. This motivates the study of the coupling of the s-wave crypto-heptaquark system to the p-wave pentaquark systems.

The simplest model to couple a s-wave crypto-heptaquark to a p-wave pentaquark consists in assuming the standard $^3P_0$ quark-antiquark creation/annihilation potential. The coupling form factor is computed with the overlap of the s-wave crypto-heptaquark $\bar{K}+N+\bar{K}$ with the p-wave pentaquarks $\bar{K}+\Sigma$ (or $\pi+\Xi$) plus a flavour singlet $^3P_0$ quark-antiquark pair. Between these two wave-functions we must sandwich the quark anti-simmetriser. After separating the center of mass coordinates, the coupling is equivalent to an overlap of harmonic oscillator wave-functions of the six Jacobi coordinates depicted in Fig. 1. The Ribeiro graphical [37] rules, and the harmonic oscillator energy conservation, require that at least one of the heptaquark coordinates should include a radial excitation. The $\bar{K}-\bar{K}$ relative coordinate is expected to be partly excited because the $\bar{K}-\bar{K}$ are repelled. If we compare our coupling with the decay of the rho into two pions we find a suppression in the coupling by a factor of 1/3. The decay width is proportional to the square of the coupling and this factor already explains the small decay width of heptaquarks which are found in all experiments to be smaller than 20 MeV. Moreover the Ribeiro rules show that overlaps with excited wave-functions are further suppressed. However, for a consistent computation, the probability for a $^3P_0$ annihilation to occur in a pentaquark remains to be consistenly studied. Nevertheless the p-wave pentaquark components should essentially not affect the mass of the crypto-heptaquark.

The relative comparison of the Θ^+ and Ξ^{--} decay widths can be estimated with a better precision than the full computation of the decay. The decay widths depend on the phase space, and they are proportional to $\mu\sqrt{2\mu E}$ in the non-relativistic case and to E^2 in the ultra-relativistic case, when the momentum is smaller that the inverse radius α. Therefore I expect that the partial decay width of the Ξ^{--} to a p-wave $\bar{K}-\Sigma$ is larger than the Θ^+ decay to a p-wave $K-N$ by a factor of 1.5, and I expect that the partial decay width of the Ξ^{--} to a p-wave $\pi-\Xi$ is larger than the Θ^+ decay to a p-wave $K-N$ by a factor smaller than 2. Thus it is expected that the total decay width of the Ξ^{--} is larger by a factor $\simeq 3$ than the decay width of the Θ^+. Nevertheless, because the decay width of the Θ^+ is quite small, the coupling of the crypto-heptaquark to the p-wave pentaquark is not expected to be large.

V. CONCLUSION

To conclude, in this paper I address the exotic anti-decuplets with a standard quark model Hamiltonian, where the quark-antiquark annihilation is constrained by the spontaneous breaking of chiral symmetry. I first derive a criterion showing that the Θ^+ and Ξ^{--} hadrons very recently discovered cannot be an s-wave or p-wave pentaquark. It is plausible that the Ξ^{--} is a linear $\bar{K}-N-\bar{K}$ molecule, a heptaquark state. The $I=0$, Θ^+ and the $I=1/2$, N^* are also heptaquarks, or linear meson-meson-baryon molecules. In these linear molecules the two external hadrons do not overlap. All these heptaquarks have a positive parity. The only pentaquark is the iso-triplet Σ^*, with the quantum numbers of a tightly bound s-wave $\bar{K}-N$ system, a negative parity system. The spectrum of the pentaquark and heptaquark antidecuplets is depicted in Figure 2, assuming that the experimental $M_\Theta = 1.54$ GeV and $M_\Xi = 1.86$ GeV are correct. Importantly, the $\Xi^{--}(1862)$ is expected to decay

both to the $\bar{K}-\Sigma$ and to the $\pi-\Xi$ p-wave channels.

For a future improvement of this work it is important to discuss the size parameter α^{-1}. In our previous paper [15] we used a quite large $\alpha \simeq 10$ Fm^{-1}, for the potential involving the pion. The pion is expected to couple with a shorter range interaction than other hadrons, because the Adler zero suppresses the low momentum part of the pionic couplings. Nevertheless the very short range pion potential and the longer range kaon potential suggest that the hadron-hadron interactions should have at least two different interactions. For instance the N-N interaction is decomposed in a short range interaction due to quark Pauli repulsion, a medium range σ or Two Pion Exchange Potential, and a long range One Pion Exchange Potential. The use of similar interactions may further increase the binding of multiquark molecules. Other technical improvements that deserve to be investigated, although they are not straightforward to implement at the microscopic level of quarks, are the coupling to p-wave channels and the exact three body equations.

I expect that the advocated heptaquarks will help to motivate the detailed experimental studies of the Θ_5, N^*, Σ^* and Ξ_5^* [38, 39, 40]. This will be crucial to discriminate the different theoretical models candidate to explain the anti-decuplet, [6, 15, 23, 24, 41, 42, 43, 44, 45, 46, 47, 48, 49].

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